

MATHEMATICAL MODELLING AND OPTIMISATION TECHNIQUES USED IN DAIRY FARMING

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Abstract

This paper aims to present the methods used so far for mathematical modelling and the optimization techniques of the dairy farming sector. Article content is based on the various papers published from 1940, a period which is considered to taken the first steps in mathematical modelling and optimization of technology used in dairy farming. In literature are described three main groups of optimization methods: intuitive methods (empirical, quantitative and qualitative analysis, graphs and charts, etc.), statistical and mathematical (clusters, dynamic series analysis, correlations, etc.) and operational research (linear programming, non-linear programming, square programming, etc.). Currently, both farmers and staff involved in livestock research, have a number of computer softwares, very affordable, allowing quick and efficient analysis of information that build mathematical models developed to optimize dairy farming technology. In addition, these programs allow maximum freedom in terms of how the mathematical models are constructed and their degree of complexity. A good example is the optimization of feed rations and recipes that can be made with specialized softwares or general softwares such as Excel. In both cases, the biological limits imposed restrictions on the model, but the human factor is the one who decides the number of feeds or the number of nutrients that make the object of optimization. Mathematical modelling and optimization activities in this area are of greatest interest because they allow, through a systemic and causal approach of the phenomena that occur in the dairy farm system, achieving economic goals such as minimizing costs, optimizing the use of available resources and maximizing income or profit.

Key words: dairy, farm system, mathematical modelling, optimisation.

INTRODUCTION

Dairy farming is an important branch of animal husbandry due to the share on that milk occupy in human nutrition and the nutritional qualities of this type of food. This statement is supported by the following figures: in the world there are about 1.5 billions cattle (for the year 2013, FAOSTAT 2015) and the milk production is about 626 milions tonnes (for the year 2012, FAOSTAT 2015), while in Romania are registered over 2 milions heads of cattle (for the year 2013, FAOSTAT 2015) and the milk production stands around of 4.33 milions tons (for the year 2012, FAOSTAT 2015). In the year of 2012, the cow milk production at global level accounted for 83% of the total milk production, and in Romania 87% (FAOSTAT, 2015).

However, over time, there have been various imbalances, regarding the relationship between supply and demand that have led to the dramatic fall of the prices offered to the dairy

farmers. This risk is imminent in the current context that requires the removal of milk quotas starting with April the 1st 2015.

In these circumstances, we consider that one of the ways that the dairy farms can remain profitable, throughout their existence, is to use mathematical modelling to identify the best optimisation techniques for the technologies that are used in dairy farm.

MATERIALS AND METHODS

This paper is a bibliographic study of the results obtained in this area in order to elaborated, in the future, a dynamic macromodel of the dairy farm that allow an integrated optimisation of the used technologies.

For this study, we used the scientific results obtained so far in this field and published in various papers (books, treatises, handbooks, PhD theses and scientific articles published in the scientific symposia dedicated to agriculture

and livestock, available in print in libraries or in digital from the scientific data bases from the internet). After identification of the papers that are of great interest for this article, we selected on those with increased applicability in practice, to be presented here. In addition, our paper contains a short presentation of the main computerized programs, specialized or not in this area, which can be used in modelling and optimisation of specific technologies for dairy cows.

The period of time studied in this work is that contained between the years 1940 and 2015.

RESULTS AND DISCUSSIONS

In the case of dairy farms, optimization is the action which seeks, by using various methods available, the most effective way of combining production factors, so that the end result is the best possible outcome in the given context. Considering these aspects, the dairy farm must be regarded as a system with the particularity that has to be seen both from an ecological perspective, but also an economical one.

Regarding the ecological side of this type of system, the dairy farm is a productive zoosystem which differs from a natural ecosystem because it assumes a much higher energy consumption, uses several sources of energy and does not account for biogeochemical natural cycles (Gruia and Păstîrnac, 1991). In addition, there are structural and functional differences in the sense that the control subsystem of the dairy farm, as a zooproductiv system, is being done by human intervention (Gruia and Păstîrnac, 1991). The bio-ecological perspective of the farm system, plays an important role in mathematical modelling performed for the system optimization, because it imposes a number of restrictions to the model. Dairy farm system is an integrated economical system because of the "natural" evolution in the market economy context and also it has feedback capacity in the sense that it respects the demand-offer principle. This means that the dairy farm economic system is in competition with other economic systems, more or less similar, so there is a need of a continue optimisation of the production processes to support the development capacity of the dairy

farm in time. Under these circumstances, the dairy farm has to be seen as an economic system that has the particularity that the optimization process is strongly influenced by the biological restrictions.

Initially, the optimisation process was very subjective, relying heavily on the experience of the dairy farm manager, but since 1940s, the mathematical modelling of the bio-productive processes and the farm's systemic approach, allowed development of the production functions which have highlighted the relation of dependence between the production factors, represented by the resources involved in the production process, and the result in the form of the finished product. Such a production function can be represented by the following mathematical expression: $Y = f(x_1, x_2, \dots, x_n)$, where the dependent variable "Y" represents the production obtained in dairy farm and the independent variables " x_1, x_2, \dots, x_n " signify the resources used.

The advantage of using production functions to optimize the production process of the dairy farms is the possibility of recognizing the best variant for combining the production factors in terms of quality and quantity.

In the field of optimizing the production, pioneers are considered Eilhart and Mitscherlich which, together with the mathematician Baule, in 1909, have defined the relationship "production factor - production" as being a nonlinear logarithmic function. (Drăgănescu, 1984).

The first specialized work which connects the exploitation of dairy cows and production functions has belonged to Jensen and was published in 1940 (Heady and Dillon, 1966).

In this paper, Jensen points out that, besides the necessity of knowing the nutritional requirements of dairy cows according to their productive potential, it is also required for us to know how the production is affected when the nutrients intake is increased or decreased (Jensen, 1940). Another problem at which the author tries to find an answer is regarding the way that the milk yield varies depending on the amount of concentrates administrated to the cows. Thus, the question is whether between milk production and level of concentrates administrated by ration is a constant relationship or a gradual one (Jensen, 1940).

Considering these issues addressed by Jensen, one can claim that this paper is the first one which introduces the concept of optimization, in its true sense, in the field of dairy cattle.

In the period after publication of the paper of Jensen, there have appeared a number of scientific works on finding an optimum (represented by the change in milk production) regarding the ratio between the quantity of grain and the quantity of forage in the ration. In this context, Heady and Dillon quotes in their book "Agricultural production functions", that was published in 1966, the experiment conducted by Huffman and Duncan in 1949. The two authors have studied the impact of replacing a quantity of TDN from hay, with an equal amount of TDN from concentrates. The result was an increase in milk production, which concluded that milk production function could be rather non-linear.

The next paper on this subject belongs to Ashe and was published in 1950. The results showed that the relationship between the input and output (regarding the milk yield) has a linear character for cows fed with a maximum of 1800 kg of grains / lactation, in the case of cows fed with a surplus of 1800 to 2700 kg of concentrate the increase in production is much smaller, while a grain intake over 2700 kg per cow has no positive effect on milk production (Heady and Dillon, 1966).

Since the 1970s, the attention of researchers began to move increasingly towards the development of the concept of optimizing the technologies for dairy farms by using production functions (Diaconescu, 1995), so, the number of papers published for the evolution of this area began to grow. In 1981, Balaine, Person and Miller published the paper entitled "Profit in Dairy Cattle functions and effect of measures of efficiency and prices" in order to define the profit function based on performances of cows, to establish the relationship between profit functions used and determine the effects that prices may have on characteristics of the functions and classification of cows in the context of the trend of using the economic efficiency achieved in farm as a selection criteria for the genetic improvement of cows. The conclusions reached by the authors are: 1) is advisable to define the economic efficiency, in the case of

dairy cows, as a linear function of income minus expenses per herd life time, because a linear function has a more closed correlation with other functions and is easier to understand, 2) the changes in relative prices have little effect on the classification of cows that are candidate for selection, 3) income variables (milk, fat and protein yield) have the highest correlation (> 0.44) with the daily profit, and expenses, defined as mastitis treatments (-0.21), herd life (0.19) and feed consumption (0.27) had the highest correlation with the profit function.

The trend in the '70s continued in the next decade, and researches on production functions have been enhanced by several researchers.

In this regard, we recall the article published by E. Groeneveld and M. Kovac in 1990, showing a calculation procedure based on several methods such as the method of least squares and systems of mixed equations, with applications in the animal genetic improvement.

A dominant feature of the use of production functions to optimize the dairy farm technologies is the fact that it focuses on achieving economic efficiency at the expense of technological aspects, so the production factors are dosed to achieve profit maximization (Diaconescu, 1995). This is eloquently supported by the work entitled "Profitability of dairy cow herd life". The authors, Congleton and King, have used a dynamic model to estimate the impact of increasing herd life on economic efficiency of the farm. So, the model estimates the economic efficiency variation on the base of the interaction between several factors, like the age of the cows, milk production, demand for labor, cost of treatment, etc.

We consider that targeting the efforts to optimize the dairy farm technologies to maximize the farm's economic efficiency, is appropriate given the argumentation made earlier in this paper, which has shown that dairy farm is primarily an economic system, being integrated in the market economy through the self-regulation mechanism which it has developed. In this context, we want to remember that the perspective which defines the dairy farm as a productive zoosystem should not be ignored, because it is the main

peculiarity of livestock and agricultural economic systems. This approach of the dairy farms can be easily understood and accepted, by both theorists and practitioners in the field of dairy farming, by showing how linear programming can be applied in the optimisation of feeding technologies for cows.

Mathematical programming is a very effective method for solving economic problems by its ability to identify the optimal solution from a set of possible solutions, so the production can achieve maximum growth under conditions of rational use of resources involved in the process (Drăgănescu and Drăgănescu, 1966). A practical problem solved by linear programming involves the following steps: 1) the identification of variables, 2) determination of the objective function, 3) the writing of objective function and 4) the highlighting of the non-negativity condition. The objective function is a linear function in the sense that the equations of the mathematical model are equations of first degree, and the goal is to maximize or minimize a process of economic, technical or biological nature.

Dairy production optimization by using the linear programming, can be done with the Simplex method that was developed by Dantzig in 1947.

In the case of dairy farms, the linear programming is typically used to optimize feed rations for cows.

In practice, regardless of the method used for finding the solution, solving the linear programming calculations is difficult and time consuming (increases the likelihood of errors). For this reason, the use of a computer program, that allow finding the optimal solution in a very short time, is highly recommended. The most accessible program of this type is the MS Excel software. Next, we present a practical example of feed ration for cows that was optimized by using the Solver, an Add-in from Excel that calculates the solutions for linear programming problems by using Simplex method.

In the Excel worksheet, the data is systematized as in the Table 1, and then the formulas are inserted as in the cell range from G4 to J7 plus the formula from the F7 cell. The next step is to select from the "Data" menu, the icon "Solver", action after which a window will opened, requesting the next information: 1) "Target cell" or objective function (the J7 cell); 2) "Equal to" or the objective function type. For this case, "Min" will be selected; 3) in the "By Changing Cell" section, the cells that correspond to the variables (F4:F6) will be selected; 4) and in the "Subject to the Constraints", the restrictions of mathematical model will be noted, which, in this case, are: $G7:I7 \geq G1:I1$; $G7:I7 \leq G2:I2$; $F5 \leq 6$; $F4 \geq 0$; $F5 \geq 0$; $F6 \geq 0$.

Table 1. The systematization of data in the worksheet and formulas used

	A	B	C	D	E	F	G	H	I	J
1					The nutritional requirements of a 650 kg cow with 20 kg of milk/day	Min.	16.3	15.3	1420	
2						Max.	19.5	16.3	1560	
3	DM (kg)	MNU	IDP (g)	Cost/kg	TYPE OF FEED	Quantity (kg)	DM (kg)	MNU	IDP (g)	COST
4	0.26	0.21	15	0,003 €	Corn silage		A4*F4	B4*F4	C4*F1	D4*F4
5	0.89	0.57	70	0,110 €	Alfalfa hay		A5*F5	B5*F5	C5*F2	D5*F5
6	0.88	1.00	120	0,170 €	Mixture of concentrated feed		A6*F6	B6*F6	C6*F3	D6*F6
7					TOTAL	F4+F5+F6	G4+G5+G6	H4+H5+H6	I4+I5+I6	J4+J5+J6

The next step consists in selecting the "Solve" button and the display of the solution, if there is one. In the case shown here, the solution is the one from Table 2. To illustrate better the importance of using such a method in dairy

farming, we can compare the cost obtained by using linear programming to optimize the feed ration with the cost of a feed ration that was optimized by the classical method of "attempt".

Table 2. The optimum solution for the feed ratio

	A	B	C	D	E	F	G	H	I	J
1					The nutritional requirements of a 650 kg cow with 20 kg of milk/day	Min.	16,3	15,3	1420	
2						Max.	19,5	16,3	1560	
3	DM (kg)	MNU	IDP (g)	Cost/kg	TYPE OF FEED	Quantity (kg)	DM (kg)	MNU	IDP (g)	COST
4	0.26	0.21	15	0.003 €	Corn silage	30,0	7,80	6,30	450	0,090 €
5	0.89	0.57	70	0.110 €	Alfalfa hay	5.3	4,68	3,00	368	0,579 €
6	0.88	1.00	120	0.170 €	Mixture of concentrated feed	6,3	5,28	6,00	720	1,020 €
7					TOTAL	41,3	17,76	15,3	1538	1,689 €

To this end, keeping the same nutritional requirements, we optimized purely technical the same feed ration (Table 3). Optimizing by "attempt" involves combining the available forages in various proportions based on the experience and the intuition of the person that optimizes the feed ratio until the nutritional requirements of the cow are satisfied.

One can easily see that there is a significant cost difference between the two feed ratios (0.152 euro per cow per day), although nutritional requirements are satisfied, both for

maintaining the vital functions as well for production.

Analyzing this situation, we can conclude that the economy obtained, thanks to applying the linear programming in zootechnical practice (optimizing the feed ratios), for the case discussed here, is of approximately 47 euros per lactation (305 days) for each cow in the farm. Based on these results, it can be stated that along with the increasing of the dairy farm size, the economy achieved increases, if these optimization techniques are used.

Table 3. Feed ratio optimized through the "attempting" method

	A	B	C	D	E	F	G	H	I	J
1					The nutritional requirements of a 650 kg cow with 20 kg of milk/day	Min.	16,3	15,3	1420	
2						Max.	19,5	16,3	1560	
3	DM (kg)	MNU	IDP (g)	Cost/kg	TYPE OF FEED	Quantity (kg)	DM (kg)	MNU	IDP (g)	COST
4	0,26	0,21	15	0,003 €	Corn silage	27.0	7.02	5.67	405	0.081 €
5	0,89	0,57	70	0,110 €	Alfalfa hay	7.5	6.68	4.28	525	0.825 €
6	0,88	1,00	120	0,170 €	Mixture of concentrated feed	5.5	4.84	5.50	660	0.935 €
7					TOTAL	37.5	18.54	15.45	1590	1.841 €

In dairy farms, linear programming can be applied to other activities, such as crop structure optimization, herd structure optimisation or to the genetic improvement technology. In genetic improvement, linear programming is a method for selection of animals that will be used for breeding, taking into account a number of restrictions on the availability of resources, marketing strategies

of the farm or the market trends (Jansen, 1984). Thus, Jansen argues in his article, "Linear Programming in Selection of Livestock", published in 1984, that the estimated performances of the animals can be integrated into objective functions (e.g. maximization of the dairy farm's profit), procedure which brings a higher benefit for the decision making process, compared to the results that may be

obtained by using the simple equations method that disregard the model's restrictions or production alternatives.

Another method for optimizing the activities in the dairy farm is by applying the graphs theory in the process of organization and planning the work of employees. This method allows the rationalization of work time through a sequential and global approach of the work process in order to optimize the total labor consumption (Iosif et al., 1984). As in the case of linear programming, the optimization calculations required for this segment, are facilitated by using computer programs. As a result of the application of this method in practice, the benefits obtained are the maximum possible reduction of the duration of the working process, the increased productivity of labor and reducing costs.

Since the 1990s, appear specialized computer programs for agriculture and livestock. The role of these programs was initially storing various information collected in the form of data banks, but the emergence of new programs, especially those for data processing has enabled a better use of them. Thus, implementation of the "Expert Systems" computer programmes allowed more efficient use of the computing technique and also the use of very effective electronic sensors in animal husbandry (Diaconescu, 1995).

The expert systems, also known as "information-based systems" were designed to solve complex problems and allow drawing conclusions and optimal management decisions based on computer processing (analysis, synthesis) of the raw data available. E. Teigenbaum from Stanford University, quoted by Șt. Diaconescu in 1995, gives the following definition of the expert systems "... a "smart" computer program, which, by using information and interface procedures, solve difficult problems that require a significant human experience for them to be resolved. The information required to solve such levels, plus the interface procedures used, are the concern, for such a model, of the best practitioners in the field."

In general, such a system is made up of several sub-systems (Figure 1).

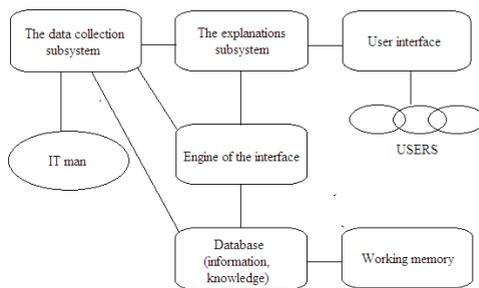


Figure 1. General scheme of an expert system (after Farnir, 1992, cited by Diaconescu, 1995)

Using expert systems, designed to optimize the exploitation of dairy cows, bring some advantages:

- reduce the cost of dairy farming by reducing the need of experts in this field;
- the expertise of an expert system is available at any time;
- can combine the expertise of several specialists in dairy farming;
- developing solutions is timely;
- the solutions are not influenced by human subjectivity;
- is a perfect source of knowledge for the farm manager and not only;
- uses complete and in an intelligent manner the available databases in the dairy farm.

The first steps in the use of expert systems in dairy farms were done by using systems of a lesser extent, in the sense that they were designed to solve specific issues or specific technological segments of dairy farming as: reproduction (Domecq et al., 1991), milking system (Hogeveen et al., 1995), forages (Patacq, 1987), animal health (Benas, 1986), the herd management (Pellerin, 1994), designing and development of dairy farms (Samer et al., 2012), veterinary medicine (Pastell and Kujala, 2007), etc.

Along with the trend of making expert systems that are specialized in the various technologies, there is the tendency of developing complex expert systems that integrate all dairy farm technologies, so that, the solutions developed by the program, to be truly integrated by taking account of the interrelations that exist between the phases of production in the dairy farms. The first steps in this direction are made by Hogeveen et al. in 1991 (Diaconescu, 1995), which creates a general scheme of the modules

and paths of a complete expert system for dairy farms (Figure 2). The functioning of these complete expert systems requires that each module elaborates specific solutions in the field that was designed for (specific algorithms for: breeding, animal health, milking technology, feeding technology, etc.) and then, by unifying these solutions, to issue findings and recommendations in the form of decisions.

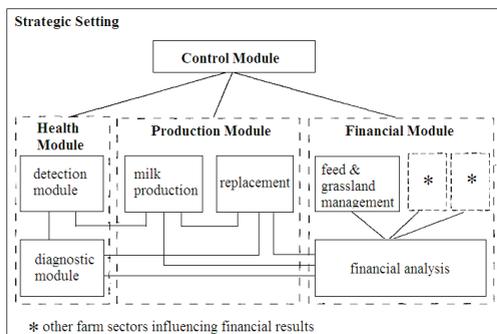


Figure 2. Search paths in a rule-based and a model-based systems (Hogeveen and all, 1991)

One of the first programs of this type is presented in the paper "DXMAS: An expert system software providing advice to dairy operators" that was published in 1993 by Schmisser and Gamroth. This program was created to identify management problems in dairy farms. As a result, it has been shown that it is able to imitate the decisions and conclusions of dairy experts in 95 management issues. It has been estimated that, by correcting these deficiencies, the dairy farmer can get an income growth ranging, at that time, between \$ 25 and \$ 450/cow/lactation (Schmisser and Gamroth, 1993).

A year later, is presented a evaluation of the LAIT-XPART VACHES program, whose expertise is based on experience of three experts in nutrition and dairy farm management. The program can calculate targets for production, costs of feeding, reproduction, etc; identify problems in feeding technology, genetic improvement, herd management, etc., based on the 950 rules and to issue findings from analysis carried out with an accuracy of 92.3% (Pellerin et al., 1994).

Starting with the year 2000, in dairy farming field occurs the concept of "Precision Feeding", which can be defined as being the optimization

of the feeding technology, so that, nutrient losses at dairy farm level to be minimized, and the pollution and costs reduced. This can be achieved by administrating rations with the real nutritional value (what the cow really consume) equal to the theoretical nutritional value, estimated by formulation (Sova et al., 2014). Thus, in the last 15 years, a series of studies have been carried out for the implementation and development of this concept (Bateman et al., 2001; Cerosaletti et al. 2004; Ternel et al., 2009; Spanghero et al., 2010; Lascono et al., 2011).

From 2010 a new trend appeared in dairy farming, that optimize the production processes by treating the cows in the herd at an individual level. This is done automatically by sensors which take and transmit in real time the information collected from the cow on: body temperature, cow's position, daily distance made by the cow, etc. The data collected is processed by computer, and the results are used to develop management strategies in the short, medium and long term (Rutten et al., 2013; Liang et al., 2013). This concept is called "Precision Livestock Farming" (André et al., 2011) and, currently, is the last optimisation technique used in dairy farming.

CONCLUSIONS

Dairy farm is primarily an economic system which has as the main feature, the fact that model's restrictions are of biological nature.

The concept of optimization of dairy farms technologies appeared in the '40s and since, it has gone through several stages: production optimization by using production functions, use of the linear programming, applying the theory of graphs, development of the expert systems and, currently, the tendency to treat dairy cows individually (Precision Dairy Farming Concept).

The objective of optimization of production in dairy farm is mainly of economic nature and secondary of a technological nature.

New studies are needed on the development and implementation of the "Precision Dairy Farming" concept in Romania, in the near future.

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