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COMPARISON OF BAYESIAN ESTIMATION AND CLASSICAL ESTIMATION OF BRUSHTOOTH LIZARDFISH (*Saurida lessepsianus* RUSSELL, GOLANI & TIKOCHINSKI 2015) GROWTH

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Abstract
The aim of this study is to compare Bayesian and Classical estimation for describing the growth curves of Brushtooth Lizardfish (Saurida lessepsianus Russell, Golani & Tikochinski 2015). Classical nonlinear regression method and Bayesian estimation method were used to obtain the estimation of the components of the von Bertalanffy growth model. The estimated parameters of the von Bertalanffy equation via two methods showed that Bayesian estimation is much better than Classical nonlinear regression in estimating growth parameters and reducing variation of growth model parameters.

Key words*: Bayesian inference, Saurida lessepsianus, Growth, von Bertalanffy.*

INTRODUCTION

In the fisheries study, fish growth data fitted by a suitable mathematical function to describe the growth, estimate parameters of growth, and compare the growth models between species or population. The shape of the growth line may change according to the genotype of living organisms, environmental factors and examined features. So, what is effect of statistical method on expression of the growth that is affected by all these factors? Answer of this question is the most important topic of finding best fitting model.
Historically, the von Bertalanffy growth equation (VBGE) has been the most common growth functions applied to fish growth in fisheries science (Ricker, 1975; Pauly, 1978; Chen et al., 1992; Helidoniotis et al., 2011). The VBGE is usually used in population dynamics and fisheries management to model individual growth of a species. The VBGE was obtained by thinking the growth of an animal because of the difference between catabolic and anabolic processes of an animal's metabolism (von Bertalanffy, 1957; Ursin, 1967; Sainsburry, 1980; Pilling et al., 2002). Generally, the VBGE is fit to length-age data using classical nonlinear least square techniques. Classical nonlinear regression assumes that there are enough measurements to say something meaningful. This somehow affects the assumptions of the VBGE. In the Bayesian approach, the data are supplemented with additional information in the form of a prior probability distribution. The prior belief about the parameters is combined with the data's likelihood function according to Bayes theorem to compute the posterior (Box and Tiao, 2011; Congdon, 2003; Siegfried and Sanso, 2006; Link and Barker, 2010; Akar and Gundogdu, 2014).
For this reason Bayesian inference provide a quantitative concept and obvious language in so as to analyze and express growth procedures. Logically, Bayesian inference is the clearest way of analyzing and interpreting growth models in light of data.
In this study we used Bayesian approach and classical approach to estimate VBGE parameters. Estimates of the von Bertalanffy growth parameters are compared with estimates Classical method. By this way, we tried to explain biological plausibility of parameters estimated by both methods.

MATERIALS AND METHODS

Length and age data of Brushtooth Lizardfish (*S. lessepsianus*) were collected between 2012 and 2013 from Iskenderun Bay of the Northeast Mediterranean Sea (Figure 1). The materials were collected by seasonally sampling using commercial bottom trawl. The fork length (FL) was measured to the nearest 1 mm. The sagittal otoliths were examined under the stereo microscope for the age determination.

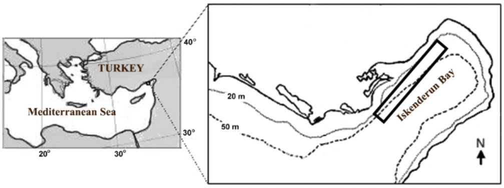

Figure 1. Study Area

Total of 400 individuals were sampled, ranging in size from 13 to 28.8 cm FL. Overall mean FL was calculated as 17.83 cm. Length-frequency distribution was given in Figure 2. As can be seen in the figure 2, the dominant length classes were 13-17 cm. Length-frequency distribution, minimum, maximum length, standard error sample size, mean length and its confidence interval values of *S. undosquamis* for each age class are listed in Table 1. As it can be seen, the age of *S. undosquamis* ranged from I to VI age classes and the most dominant age class was 2 with a value of 35.2% and age class 3 ranks second with a value of 25.2%.

The form of VBGE described by many researchers is the following;

$$L_t = L_\infty\left(1 - e^{-K(t-t_0)}\right) + \varepsilon_t$$

where t_0, L_∞ and K are the VBGE parameters, and the ε_t are assumed to be normally distributed error. The growth parameters t_0, L_∞ and K were estimated using the Classical Least Squares Method as recommended by Sparre and Venema, (1998). This produced least-squares estimates of the three von Bertalanffy growth parameters.

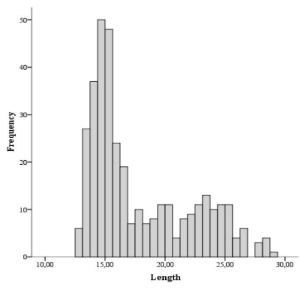

Figure 2. Length-frequency distribution

Table 1. Length-frequency distribution, minimum, maximum, standard error, 95% confidence interval and mean fork length values for each age class for Brushtooth Lizardfish

Age Class	N	Mean	Std. Error	95% Confidence Interval		Minimum	Maximum
				Lower	Upper		
1	80	13.71	0.04	13.63	13.81	13	14.2
2	141	15.29	0.05	15.17	15.41	14.4	16.8
3	101	20.53	0.21	20.11	20.95	17	23.9
4	32	24.47	0.06	24.34	24.61	24	25
5	28	25.91	0.14	25.61	26.21	25.1	26.8
6	18	28.26	0.18	27.83	28.69	27.5	28.8
Total	400	17.83	0.22	17.39	18.27	13	28.8

Bayesian approach fits the VBGE to the length at age data using Markov Chain Monte Carlo (MCMC) methods (Hastings, 1970; Gelman et al., 2004). These methods has four steps; i) Finding likelihood of the data, ii) Defining priors for all parameters, iii) Defining conditional probabilities for all parameters, and iv) Using the Bayesian method to estimating the posterior distribution for parameters (Gelman et al. 2004; Siegfried and Sanso 2006). Thus, our likelihood is as in the following:

$$L_t \sim dnormal(\mu_i, \tau)$$
$$\mu_i = L_\infty\left(1 - e^{-K(t-t_0)}\right)$$

We used informative priors for t_0, L_∞ and K based on published estimates of the same parameters in the FishBase (Froese and Pauly 2012) for *S. lessepsianus*

$$L_\infty \sim normal(40{,}64.10)$$
$$-(t_0) \sim gamma(0{,}265.10)$$
$$K \sim gamma(0{,}83.10)$$

We used uninformative priors for τ, giving the full power of estimation to the data:

$$\tau \sim gamma(0.0001, 0.0001)$$

OpenBUGS (**B**ayesian inference **U**sing **G**ibbs **S**ampling; Thomas et al., 1992; Spiegelhalter et al. 2007; http://www.openbugs.net) was used to fit the model. The estimates of parameters were evaluated based on 1000000 samples, from Markov chain Monte Carlo (MCMC) simulation of the joint posterior distribution. We used a burn-in period of 10000 chains and generated posteriors for the parameters of the VBGE with the remaining chains.

RESULTS AND DISCUSSIONS

The growth parameters calculated by classical nonlinear regression were L_∞: 48.86 cm, *K*: 0.107 $year^{-1}$ and t_0: -1.733 year (Table 2). The back-calculated lengths were determined by using von Bertalanffy growth parameters and both the observed and calculated growths in fork length are listed in Table 3. Growth curve was fitted to lengths-age for *S. lessepsianus* is showed in Figure 3. The growth parameters calculated by Bayesian nonlinear regression were L_∞: 30.62 cm, *K*: 0.3086 $year^{-1}$ and t_0: -0.3046 year (Table 2). Growth curve was fitted to lengths-age for *S. lessepsianus* is showed in Figure 3.

Table 2. Parameter estimations of both methods

Method	Parameter	Mean	Std dev	Credible Interval	
				%2.5	%97.5
Bayesian Approach	K	0.3086	0.0880	0.1724	0.4599
	L_∞	30.620	5.2520	25.160	40.000
	t_0	-0.3046	0.2796	-0.8674	-0.0051
	$1/\sigma^2$	80.500	2.9500	24.460	82.400
	σ^2	0.0121	0.0195	0.0012	0.0408
Classical Approach	L_∞	48.868	11.757	25.763	71.792
	K	0.107	0.035	0.017	0.156
	t_0	-1.733	-1.719	-3.135	-1.103
	RSE	2.800	0.851	-	-

Estimates of L_∞ of Bayesian approach for *S. lessepsianus* were much closer to observed maximum length than Classical approach. The coefficient of K in result of both approaches was between 0 and 1. However estimate K of Bayesian approach higher than Classical approach and estimate of t_0 of Bayesian approach much closer to zero than Classical approach. As we see in Fig. 3 Growth is fast until the age class II and with growth in length is slightly reduced beyond the age class II. Correlation between parameters was found $cor(L_\infty, K) = -0.78$, $cor(L_\infty, t_0) = -0.18$ and $cor(K, t_0) = 0.57$ in Bayesian approach. Note that K and t_0 decrease as L_∞ increases.

Performance of the Bayesian growth model was verified based on 1000000 samples generated from MCMC simulation. A "burn in" sample of 10000 was initially rejected and the remaining 1000000 iterations from the chain sequence thinned at a rate of 1 sample in 10 (for removing autocorrelation of MCMC). Autocorrelation of the chain diminished after a lag of about 10. Since shape posterior density affects interpretation of parameters, it is necessary to take into account of shape of density graphs (Box and Tiao, 2011). The posterior distribution plots of each parameter given in Figure 4, L_∞ and K is distributed symmetrically around mean, estimates of t_0 close to zero with a long tail to the left.

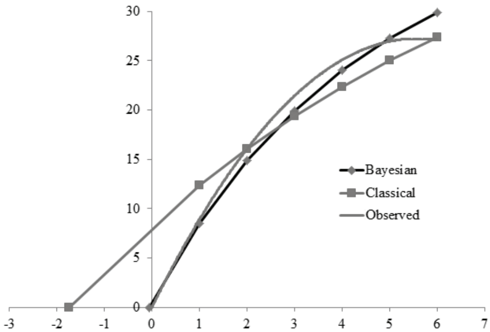

Figure 3. Bayesian and Classical von Bertalanffy length-at-age growth curve for *S. lessepsianus*

Since the growth function may vary, the shape of the growth curve for fishes may vary between populations or species also. Therefore it is essential to assess the goodness of fit in any comparison among approaches. The biggest difference in growth curve between this two approaches and the result of the estimation of VBGE parameters indicate that Bayesian approach biologically more plausible than Classical approach. According to many authors (Pauly, 1978; Chen et al., 1992; Sparre and Venema, 1998), L_∞ should be reasonably close to the maximum fish length in observed data, t_0 should be smaller or equal to zero and, K might vary between 0 and 1.

Results of Bayesian approach shows that estimate of L_∞ much more closely than Classical approach to maximum observed length (28.8 cm) and estimate of t_0 almost equal to zero (Table 2, Figure 4).

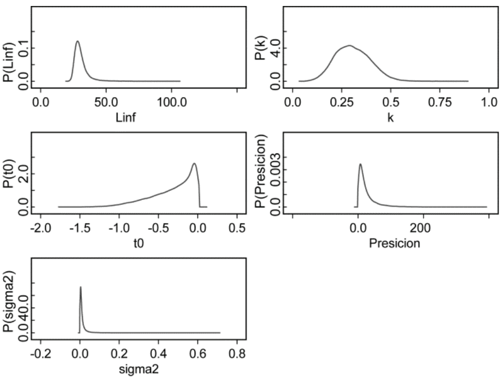

Figure 4. Kernel density histogram for the von Bertalanffy growth parameters L_∞, K, t_0, variance and precision and drawn from 100000 MCMC samples of the Bayesian brushtooth lizardfish growth model

When it is compared Bayesian estimation to previous studies, in this study, Bayesian approach produce biologically more plausible estimations (Table 3).

Table 3. Growth parameter estimates of *Saurida lessepsianus* from previous study

L_∞(cm)	K(year^{-1})	t_0(year)	Autor(s)
22.43	0.597	-1.365	Türeli and Erdem (1997)
41.27	0.118	-1.895	Çiçek (2006)
42	0.51	-0.29	Gökçe et al. (2007)
38.05	0.124	-1.680	Çiçek and Avşar (2011)
41.57	0.118	-1.895	Manaşırlı et al. (2011)
30.62	0.308	-0.304	This Study (Bayesian)
48.86	0.086	-2.119	This Study (Classical)

According to Pauly, (1978) and Sparre and Venema, (1998), estimated parameters of VBGE should be related to biological characteristics of inspected species. Since *S. lessepsianus* is a demersal fish, it is necessary to have lower K value. As it can be seen in Table 3, there are no similarities for the estimated K values between this study and reported by other studies from Turkish coast. When we consider the estimated value of t_0, in this study, Bayesian estimation of t_0 much closer to 0 than all other previous studies given in Table 3. It is identified with the nature of Bayes theory. Since Bayesian approach takes into account of prior knowledge, this minimizes difference between real value and estimation (Box and Tiao, 2011; Lee, 2004). According to Sparre and Venema, (1998), because of the growth begins at hatching when the larva already has a certain length, biologically, t_0 has no meaning. For this reason, value of t_0 should be zero or so.

The usage of prior knowledge in Bayesian approach may have effect on precision. When we consider that precision parameter shows the possibility of deviation of estimation from real value, low variance of Bayesian approach (0.01) makes it more preferable than classical approach (Helser and Lai, 2004; Helser et al., 2007). We can also compare with Bayesian and classical approach with considering variance. As it can be seen in Table 2 variance of Bayesian approach is lower than Classical approach.

Since growth has correlation to reproduction and survival of fish and its wide usage in fisheries population dynamics, it is one of the most important life history traits of fishes (Beverton and Holt, 1957; Ricker, 1975; Beverton, 1992; Helser and Lai, 2004). Von Bertalanffy growth model is most often estimated using the Classical approach. Comparatively fewer studies have reported quantitative comparisons of various statistical procedures. Generally, this is because popularity and simplicity of statistical methods. The methods presented in this study based on a nonlinear Bayesian growth model of brush tooth lizard fish growth clearly demonstrate that analysis of two methods simultaneously is not a limitation as discussed below.

CONCLUSIONS

Main conclusion from this study of Brushtooth Lizardfish growth is that suitable statistical methods can be used to assess growth parameters. Our use of nonlinear analysis using Bayesian inference for fish growth has the advantage of biological plausibility. If new sample are collected in the future, results of this study can provide an informative and plausible prior.

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